Pensieve Header: Wheeled Semi-Symmetrized calculus in the 2D quotient: Solving for the “conj” coefficients, II.

In[1]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\w-Computations"]
```

Out[1]=
```
C:\drorbn\AcademicPensieve\Projects\w-Computations
```

In[2]:=
```
ar[i_, j_] := t[i] h[j]
```

In[71]:=
```
μCollect[μ_] := Collect[μ, _h, Collect[#, _t, FullSimplify] &];
SetAttributes[μForm, Listable];
μForm[μ_] := Module[
  {tails, heads, mat},
  tails = Union[Cases[μ, t[s_] :→ s, Infinity]];
  heads = Union[Cases[μ, h[s_] :→ s, Infinity]];
  mat = Outer[Coefficient[μ, h[#1] t[#2]] &, heads, tails];
  PrependTo[mat, t /@ tails];
  mat = Prepend[Transpose[mat],
    Prepend[h /@ heads, μ /. (h[_] | t[_]) → 0]
   ];
  MatrixForm[mat]
 ]
```

In[5]:=
```
hm[x_, y_, z_][μ_] := Module[
  {ξ, η},
  ξ = D[μ, h[x]];
  η = D[μ, h[y]];
  μCollect[(μ /. h[x | y] → 0) + ξ h[z] + (1 + ξ /. t[i_] :→ c[i]) η h[z]]
 ]
```

In[6]:=
```
hm[3, 4, 5][ar[1, 3] + ar[2, 4]]
```

Out[6]=
```
h[5] (t[1] + (1 + c[1]) t[2])
```

In[7]:=
```
hm[3, 4, 5][ar[1, 3] + ar[2, 4]] // μForm
```

Out[7]//MatrixForm=

$$\begin{pmatrix} 0 & h[5] \\ t[1] & 1 \\ t[2] & 1+c[1] \end{pmatrix}$$

In[8]:=
```
μ1 = ar[1, 4] + ar[2, 5] + ar[3, 6]
```

Out[8]=
```
h[4] t[1] + h[5] t[2] + h[6] t[3]
```

In[9]:=
```
μ1 // μForm
```

Out[9]//MatrixForm=

$$\begin{pmatrix} 0 & h[4] & h[5] & h[6] \\ t[1] & 1 & 0 & 0 \\ t[2] & 0 & 1 & 0 \\ t[3] & 0 & 0 & 1 \end{pmatrix}$$

In[10]:=
```
hm[4, 5, 7][μ1]
```

Out[10]=
```
h[7] (t[1] + (1 + c[1]) t[2]) + h[6] t[3]
```

In[11]:=
```
hm[7, 6, 8][hm[4, 5, 7][μ1]]
```

Out[11]=
```
h[8] (t[1] + (1 + c[1]) t[2] + (1 + c[1]) (1 + c[2]) t[3])
```

In[12]:=
```
hm[4, 7, 8][hm[5, 6, 7][μ1]] // μForm
```

Out[12]//MatrixForm=

$$\begin{pmatrix} 0 & h[8] \\ t[1] & 1 \\ t[2] & 1+c[1] \\ t[3] & (1+c[1])\,(1+c[2]) \end{pmatrix}$$

In[13]:=
```
hm[7, 6, 8][hm[4, 5, 7][μ1]] - hm[4, 7, 8][hm[5, 6, 7][μ1]]
```

Out[13]= 0

In[14]:=
```
hfac[z_, xtails_List → x_, y_][μ_] := Module[
    {ytails},
    ytails = Complement[
      Union[Cases[μ, t[s_] :→ s, Infinity]],
      xtails
     ];
    hfac[z, xtails → x, ytails → y][μ]
   ];
hfac[z_, x_,  ytails_List → y_][μ_] := Module[
    {xtails},
    xtails = Complement[
      Union[Cases[μ, t[s_] :→ s, Infinity]],
      ytails
     ];
    hfac[z, xtails → x, ytails → y][μ]
   ];
hfac[z_, xtails_List → x_, ytails_List → y_][μ_] := Module[
   {ξ, ξ, η},
   ξ = D[μ, h[z]];
   ξ = ξ /. ((t[#] → 0) & /@ ytails);
   η = ξ /. ((t[#] → 0) & /@ xtails);
   μCollect[μ - h[z] ξ + h[x] ξ + h[y] η / (1 + ξ /. t[s_] → c[s])]
  ]
```

In[17]:=
```
hm[3, 4, 5][ar[1, 3] + ar[2, 4]] // μForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0 & h[5] \\ t[1] & 1 \\ t[2] & 1+c[1] \end{pmatrix}$$

In[18]:=
```
hm[3, 4, 5][ar[1, 3] + ar[2, 4]] // hfac[5, {1} → 3, 4]
```

Out[18]= h[3] t[1] + h[4] t[2]

- **conj2 is designed to work only in the feedback case.**

In[96]:=
```
t[y] → (a1 + a2 c[y]) t[y] / (1 + γ (a3 + a4 c[y]))
```

Out[96]= $t[y] \to \dfrac{(a1 + a2\,c[y])\,t[y]}{1 + \gamma\,(a3 + a4\,c[y])}$

```
conj2[y_, x_][μ_] := Module[
   {γ},
   γ = Coefficient[μ, ar[y, x]];
   μCollect[μ /. t[y] → (a1 + a2 c[y]) γ t[y] / (1 + γ (a3 + a4 c[y]))]
  ];
```

In[99]:=
```
Riffle[
 ComposeList[
   ops = {conj2[1, 2], conj2[1, 3], hm[2, 3, 2]},
   μ3 = 0 α1 ar[1, 1] + α2 ar[2, 1] + α3 ar[1, 2] + α4 ar[1, 3]
  ] // μForm,
 ops
]
```

$$\left\{\begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & 0 & \alpha_3 & \alpha_4 \\ t[2] & \alpha_2 & 0 & 0 \end{pmatrix}, \text{conj2}[1,2], \begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & 0 & \frac{(a1+a2\,c[1])\,\alpha_3}{1+(a3+a4\,c[1])\,\alpha_3} & \frac{(a1+a2\,c[1])\,\alpha_4}{1+(a3+a4\,c[1])\,\alpha_3} \\ t[2] & \alpha_2 & 0 & 0 \end{pmatrix},\right.$$

$$\text{conj2}[1,3], \begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & 0 & \frac{(a1+a2\,c[1])^2\,\alpha_3}{1+(a3+a4\,c[1])\,(\alpha_3+(a1+a2\,c[1])\,\alpha_4)} & \frac{(a1+a2\,c[1])^2\,\alpha_4}{1+(a3+a4\,c[1])\,(\alpha_3+(a1+a2\,c[1])\,\alpha_4)} \\ t[2] & \alpha_2 & 0 & 0 \end{pmatrix},$$

$$\left.\text{hm}[2,3,2], \begin{pmatrix} 0 & h[1] & h[2] \\ t[1] & 0 & \frac{(a1+a2\,c[1])^2\left(\alpha_3+\alpha_4\left(1+\frac{c[1]\,(a1+a2\,c[1])^2\,\alpha_3}{1+(a3+a4\,c[1])\,(\alpha_3+(a1+a2\,c[1])\,\alpha_4)}\right)\right)}{1+(a3+a4\,c[1])\,(\alpha_3+(a1+a2\,c[1])\,\alpha_4)} \\ t[2] & \alpha_2 & 0 \end{pmatrix}\right\}$$

In[100]:=
```
Riffle[
 ComposeList[
   ops = {hm[2, 3, 2], conj2[1, 2]},
   μ3
  ] // μForm,
 ops
]
```

Out[100]=

$$\left\{\begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & 0 & \alpha_3 & \alpha_4 \\ t[2] & \alpha_2 & 0 & 0 \end{pmatrix}, \text{hm}[2,3,2], \begin{pmatrix} 0 & h[1] & h[2] \\ t[1] & 0 & \alpha_4+\alpha_3\,(1+c[1]\,\alpha_4) \\ t[2] & \alpha_2 & 0 \end{pmatrix},\right.$$

$$\left.\text{conj2}[1,2], \begin{pmatrix} 0 & h[1] & h[2] \\ t[1] & 0 & \frac{(a1+a2\,c[1])\,(\alpha_4+\alpha_3\,(1+c[1]\,\alpha_4))}{1+(a3+a4\,c[1])\,(\alpha_4+\alpha_3\,(1+c[1]\,\alpha_4))} \\ t[2] & \alpha_2 & 0 \end{pmatrix}\right\}$$

In[101]:=
```
{c2l = μ3 // conj2[1, 2] // conj2[1, 3] // hm[2, 3, 2],
 c2r = μ3 // hm[2, 3, 2] // conj2[1, 2]}
```

Out[101]=

$$\left\{\left((a1+a2\,c[1])^2\,h[2]\left(\alpha_3+\alpha_4\left(1+\frac{c[1]\,(a1+a2\,c[1])^2\,\alpha_3}{1+(a3+a4\,c[1])\,(\alpha_3+(a1+a2\,c[1])\,\alpha_4)}\right)\right)t[1]\right)\Big/\right.$$
$$(1+(a3+a4\,c[1])\,(\alpha_3+(a1+a2\,c[1])\,\alpha_4))+h[1]\,\alpha_2\,t[2],$$
$$\left.\frac{(a1+a2\,c[1])\,h[2]\,(\alpha_4+\alpha_3\,(1+c[1]\,\alpha_4))\,t[1]}{1+(a3+a4\,c[1])\,(\alpha_4+\alpha_3\,(1+c[1]\,\alpha_4))}+h[1]\,\alpha_2\,t[2]\right\}$$

In[102]:=
```
SolveAlways[c2l == c2r, {t[1], t[2], h[1], h[2], α1, α2, α3, α4, c[1]}]
```

Out[102]=
```
$Aborted
```

In[94]:=
```
μForm /@ ({c2l, c2r} /. {a1 → 1, a2 → 0})
```

Out[94]=

$$\left\{\begin{pmatrix} 0 & h[1] & h[2] \\ t[1] & \frac{\alpha_1}{1+(a3+a4\,c[1])\,(\alpha_3+\alpha_4)} & \frac{\alpha_3+\alpha_4\left(1+\frac{c[1]\,\alpha_3}{1+(a3+a4\,c[1])\,(\alpha_3+\alpha_4)}\right)}{1+(a3+a4\,c[1])\,(\alpha_3+\alpha_4)} \\ t[2] & \alpha_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & h[1] & h[2] \\ t[1] & \frac{\alpha_1}{1+(a3+a4\,c[1])\,(\alpha_4+\alpha_3\,(1+c[1]\,\alpha_4))} & \frac{\alpha_4+\alpha_3\,(1+c[1]\,\alpha_4)}{1+(a3+a4\,c[1])\,(\alpha_4+\alpha_3\,(1+c[1]\,\alpha_4))} \\ t[2] & \alpha_2 & 0 \end{pmatrix}\right\}$$

In[95]:=
```
SolveAlways[c2l == c2r /. {a1 → 1, a2 → 0},
  {t[1], t[2], h[1], h[2], α1, α2, α3, α4, c[1]}]
```

Out[95]= {{a3 → 0, a4 → 0}}